

**FOR OFFICIAL USE**

Score for accuracy	×	Mult. factor for speed	=			
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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特別聲明，答案須用數字表達，並化至最簡。

1. There are 100 light bulbs labeled from 1 to 100, and there are 100 students in the class. Each student takes a turn to press the switch buttons of the light bulbs with a label that is a multiple of their assigned number. For example, the first student presses the switch buttons of the light bulb with label 1 and all of its multiples, the second student presses the switch buttons of the light bulb with label 2 and all of its multiples, and so on. Each student will only come out once, and if a light bulb is on, it becomes off after being pressed, and vice versa. All the light bulbs are off at the beginning.  $X$  is the number of light bulbs that are on after the 100th student presses. Find the value of  $X$ .

有 100 個燈泡，編號從 1 到 100。班上有 100 名學生。每個學生輪流按下燈泡開關，程序如下：第一個學生按下編號為 1 及其倍數的燈泡開關，第二個學生按下編號為 2 及其倍數的燈泡開關，以此類推。每個學生只出來一次。如果燈泡亮著，按下開關後就會熄滅，反之亦然。一開始所有燈泡都是熄滅的。 $X$  代表在第 100 個學生按下開關後，燈泡亮著的數量。求  $X$  的值。

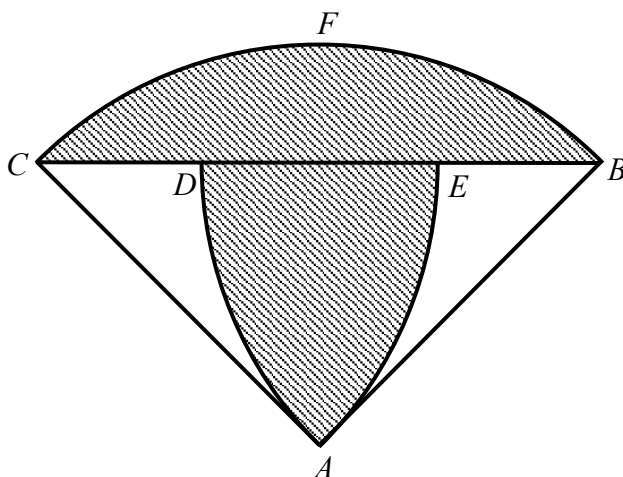
$X =$

2. Given  $x + \frac{1}{x} = 2\sqrt{5}$ . Find  $x^5 - \frac{1}{x^5}$ .

已知  $x + \frac{1}{x} = 2\sqrt{5}$ 。求  $x^5 - \frac{1}{x^5}$ 。

3. In the following figure,  $ABC$  is an isosceles right triangle with  $AB = 2$  and a right angle at  $A$ . The figure includes three arcs: arc  $BFC$ , arc  $AD$ , and arc  $AE$ . Arc  $BFE$  has a radius of  $AB$  and is drawn from centre  $A$ . Arc  $AD$  is drawn from centre  $B$  with radius  $AB$ , while arc  $AE$  is drawn from centre  $C$  with radius  $AC$ . The shaded region of the following figure, formed by the isosceles right triangle and the arcs, needs to be determined. Find the area of this shaded region. (Use  $\pi = 3$ )

下圖中， $ABC$  是一個等腰直角三角形及其角  $A$  為直角， $AB = 2$ 。圖中有三個弧，它們分別是弧  $BFC$ 、弧  $AD$  和弧  $AE$ 。弧  $BFE$  是以  $A$  為圓心、 $AB$  為半徑畫出的。弧  $AD$  是以  $B$  為圓心、 $AB$  為半徑畫出的。弧  $AE$  是以  $C$  為圓心、 $AC$  為半徑畫出的。這個等腰直角三角形和這三個弧組成了以下的圖形。請計算這個圖形的陰影部分面積。(請用  $\pi = 3$ )




4. Using the sequence of positive integers 1, 2, 3, 4, 5, 6, and so on, a new integer is formed by concatenating them: 123456789101112131415161718... The leftmost digit in this integer is defined as first digit. What is the digit at position 2023?

使用正整數序列 1、2、3、4、5、6 等等，通過將它們連接起來形成一個新的整數：123456789101112131415161718... 這個整數的最左邊的數位被定義為第一個數位。問在第 2023 數位是 0 至 9 的哪一個數？

**Hong Kong Mathematics Olympiad (2022/23)**  
**Finals (Group – Event 2)**

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			+	Bonus Score		<input type="text"/>	Time	<input type="text"/>
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			Total Score			<input type="text"/>	Min.	Sec.

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Find the minimum value of  $x^3 + y^3$  if  $x$  and  $y$  are two positive integers whose sum is 15.

假如  $x$  和  $y$  都是正整數且它們的和是 15，找出  $x^3 + y^3$  的最小值。

2. A cubic dice has faces marked with numbers from 6 to 11. The dice was rolled twice. At the first time, the sum of the numbers on the four lateral faces was 36. At the second time, the sum was 33. What number is on the face opposite to the one with the number 10?

有一顆骰子，它的六個面上分別寫上數字 6 至 11。現投擲這顆骰子兩次，第一次得知四個側面的數字和是 36，第二次的數字和是 33。請問數字 10 的對面是甚麼數字？

3. Find the greatest common divisor of  $10^{12} + 809$  and  $10^{10} + 8$ .

找出  $10^{12} + 809$  和  $10^{10} + 8$  的最大公因數。

4. Hong Kong is located at  $(0, 0)$  of a grid map and a typhoon is at  $(4, -2)$ . Suppose the typhoon will only move to the west (left) with a probability of 0.1 or to the north (up) with a probability of 0.9, and may only change course after moving one unit distance. What is the probability that it will hit Hong Kong? (Give your answer in 4 significant figures.)

在直角坐標平面上，香港的坐標是  $(0, 0)$ ，颱風是  $(4, -2)$ 。假設颱風向西（左）移動時，概率為 0.1，和向北（上）移動時，概率為 0.9，而且只能在移動一個單位距離後才可改變方向，請問這個颱風遇到香港的概率是多少？（答案需準確至四位有效數字。）

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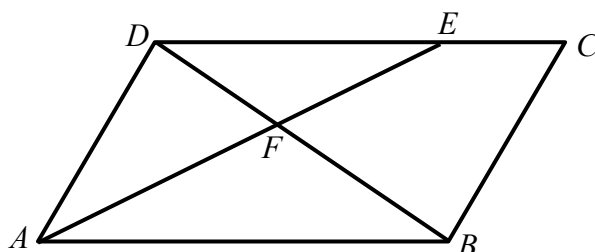
1. Let  $a_n$  be a sequence such that  $a_n = \frac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ . Find the value of  $s$  where  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ .

設  $a_n$  為序列且  $a_n = \frac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ 。如果  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ ，求  $s$  的值。

$s =$

2. Let  $ABCD$  be a parallelogram with  $AB = 40$ ,  $AD = 24$  and  $DB = 56$ . The angle bisector of  $\angle DAB$  meets side  $DC$  at the point  $E$ , and the diagonal  $DB$  meets  $AE$  at the point  $F$ . If the area of  $ABCD$  is 560 square units, find the area of the quadrilateral  $ECBF$ .

設  $ABCD$  為平行四邊形且  $AB = 40$ ， $AD = 24$  及  $DB = 56$ 。 $\angle DAB$  的角平分線與  $DC$  相交於  $E$  點，且對角線  $DB$  與  $AE$  相交於  $F$  點。求四邊形  $ECBF$  的面積。



3. Let  $f(x)$  be a function such that

$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right), \quad x \neq 0, 1$$

Find the value of  $f(-1)$ .

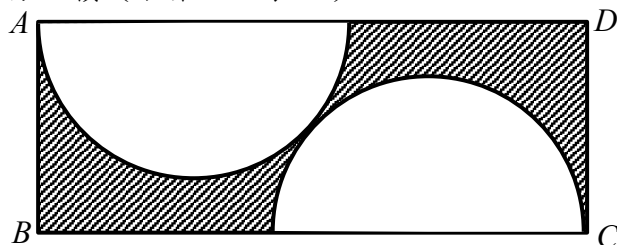
設  $f(x)$  為函數並滿足

$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right), \quad x \neq 0, 1$$

求  $f(-1)$  的值。

4. In the following figure,  $ABCD$  is a rectangle. The two semi-circles are identical and they are tangent to each other. If  $AB = 2$  and  $BC = 6$ , find the area of the shaded part in terms of  $\pi$ .

下圖中， $ABCD$  是一個長方形。兩個半圓形完全相等且它們彼此相切。如果  $AB = 2$  及  $BC = 6$ ，求圖中陰影部分面積（答案以  $\pi$  表示）。



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除非特別聲明，答案須用數字表達，並化至最簡。

1. Find the product of all the real roots of the equation  $x^{\log_{10} x} = 10$ .

求方程  $x^{\log_{10} x} = 10$  所有實根的積。

2. Let  $p$  be a prime and  $m$  be an integer. If  $p(p+m)+2p=(m+2)^3$ , find the greatest possible value of  $m$ .

設  $p$  為質數及  $m$  為整數。如果  $p(p+m)+2p=(m+2)^3$ ，求  $m$  的最大值。

3. If the length of one side of a regular tetrahedron is 1, find the volume of such tetrahedron.

如果正四面體的邊長是 1，求該正四面體的體積。

4. Let  $P$  be the product of 3659893456789325678 and 342973489379256. Find the number of digits of  $P$ .

設  $P$  為 3659893456789325678 和 342973489379256 的乘積。求  $P$  的位數。